1-4 Solving Equations

Common Core State Standards

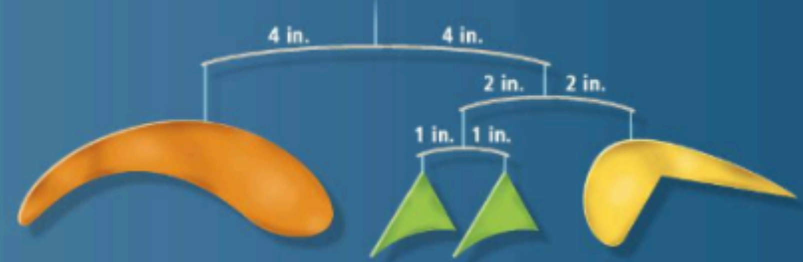
A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. **Also A-CED.A.4**

MP 1, MP 2, MP 3, MP 4, MP 6

Objectives To solve equations
To solve problems by writing equations

SOLVE IT! Getting Ready!

You bought a mobile kit. You read on the package that the weight of the entire mobile is 40 oz. Each inch of crossbar weighs 1 oz. What is the weight of each shape? Justify your reasoning.



Lesson Vocabulary
- equation
- solution of an equation
- inverse operations
- identity
- literal equation

An **equation** is a statement that two expressions are equal. In this lesson you will use equations to model and solve problems.

Essential Understanding You can use the properties of equality and inverse operations to solve equations.

take note **Properties** **Properties of Equality**

Assume a, b, and c represent real numbers.

Property	Definition	Example
Reflexive	$a = a$	$5 = 5$
Symmetric	If $a = b$, then $b = a$.	If $\frac{1}{2} = 0.5$, then $0.5 = \frac{1}{2}$.
Transitive	If $a = b$ and $b = c$, then $a = c$.	If $2.5 = 2\frac{1}{2}$ and $2\frac{1}{2} = \frac{5}{2}$, then $2.5 = \frac{5}{2}$.
Substitution	If $a = b$, then you can replace a with b and vice versa.	If $a = b$ and $9 + a = 15$, then $9 + b = 15$.

Properties — Properties of Equality, Continued

take note

Assume *a*, *b*, and *c* represent real numbers.

Property	Definition	Example
Addition	If $a = b$, then $a + c = b + c$.	If $x = 12$, then $x + 3 = 12 + 3$.
Subtraction	If $a = b$, then $a - c = b - c$.	If $x = 12$, then $x - 3 = 12 - 3$.
Multiplication	If $a = b$, then $a \cdot c = b \cdot c$.	If $x = 12$, then $x \cdot 3 = 12 \cdot 3$.
Division	If $a = b$, then $a \div c = b \div c$ (with $c \neq 0$).	If $x = 12$, then $x \div 3 = 12 \div 3$.

Solving an equation that contains a variable means finding all values of the variable that make the equation true. Such a value is a **solution of the equation**. To find a solution, isolate the variable on one side of the equation using *inverse operations*.

Inverse operations are operations that "undo" each other. Addition and subtraction have this inverse relationship, as do multiplication and division.

Problem 1 Solving a One-Step Equation

Plan

How can you isolate the variable?
To isolate the variable, you have to remove the +4 from the left side of the equation.

What is the solution of $x + 4 = -12$?

$x + 4 = -12$

$x + 4 - 4 = -12 - 4$ Subtraction Property of Equality

$x = -16$ Simplify.

Subtraction is the inverse operation of addition, so subtract 4 from each side.

Check $-16 + 4 \stackrel{?}{=} -12$

$-12 = -12$ ✔

Got It? **1.** What is the solution of $12b = 18$?

Problem 2 Solving a Multi-Step Equation

Plan

How do you solve an equation with the variable on both sides?
Choose a side for the variable and remove it from the other side.

What is the solution of $-27 + 6y = 3(y - 3)$?

$-27 + 6y = 3(y - 3)$

$-27 + 6y = 3y - 9$ Distributive Property

$6y = 3y + 18$ Add 27 to each side.

$3y = 18$ Subtract $3y$ from each side.

$y = 6$ Divide each side by 3.

GRIDDED RESPONSE

Got It? **2.** What is the solution of $3(2x - 1) - 2(3x + 4) = 11x$?

Problem 3 Using an Equation to Solve a Problem

Flowers "Flower carpets" incorporate hundreds of thousands of brightly-colored flowers as well as grass, tree bark, and sometimes fountains to form intricate designs and motifs. The flower carpet shown here, from Grand Place in Brussels, Belgium, has a perimeter of 200 meters. What are the dimensions of the flower carpet?

Plan

How can you relate the dimensions to the perimeter?
Use the formula for the perimeter of a rectangle.

Relate 2 • width plus 2 • length equals perimeter

Define Let x = the width.

Then $3x$ = the length.

Write $2 \cdot x + 2 \cdot 3x = 200$

$2x + 2 \cdot 3x = 200$

$2x + 6x = 200$ Multiply.

$8x = 200$ Combine like terms.

$\frac{8x}{8} = \frac{200}{8}$ Divide each side by 8.

$x = 25$ Simplify.

Find the length: $3x = 3 \cdot 25 = 75$.

The width is 25 meters. The length is 75 meters.

Got It? 3. Suppose the flower carpet from Problem 3 had a perimeter of 320 meters. What would the dimensions of the flower carpet be?

An equation does not always have one solution. An equation has no solution if no value of the variable makes the equation true. An equation that is true for every value of the variable is an **identity**.

Essential Understanding Sometimes, no value of the variable makes an equation true. For identities, all values of the variable make the equation true.

Think

What does it mean for an equation to be sometimes true?
An equation is sometimes true if it is true for some, but not all, values of the variable.

Problem 4 Equations With No Solution and Identities

Is the equation *always, sometimes,* or *never* true?

A $11 + 3x - 7 = 6x + 5 - 3x$

$4 + 3x = 3x + 5$

$4 = 5$ Never true!

The last equation is not true, so no value of x makes the first two equations true. The original equation has no solution. It is never true.

B $6x + 5 - 2x = 4 + 4x + 1$

$4x + 5 = 4x + 5$

$4x = 4x$

$0 = 0$ ✔ Always true!

The last equation is true, so any value of x makes the first three equations true. The original equation is always true. It is an identity.

Got It? 4. Is the equation *always, sometimes,* or *never* true?

a. $7x + 6 - 4x = 12 + 3x - 8$ b. $2x + 3(x - 4) = 2(2x - 6) + x$

A **literal equation** is an equation that uses at least two different letters as variables. You can solve a literal equation for any one of its variables by using the properties of equality. You solve for a variable "in terms of" the other variables.

Plan

How do you solve a literal equation for one of its variables?
Use inverse operations to isolate the indicated variable.

Problem 5 Solving a Literal Equation

The equation $C = \frac{5}{9}(F - 32)$ relates temperatures in degrees Fahrenheit F and degrees Celsius C. What is F in terms of C?

$C = \frac{5}{9}(F - 32)$

$\frac{9}{5}C = F - 32$ Multiply each side by $\frac{9}{5}$.

$\frac{9}{5}C + 32 = F$ Add 32 to each side.

$F = \frac{9}{5}C + 32$ Symmetric Property

Got It? 5. a. The equation $K = C + 273$ relates temperatures kelvins K and degrees Celsius C. What is C in terms of K?

b. **Reasoning** Is the equation relating temperatures in kelvins and degrees Celsius *always, sometimes,* or *never* true? Explain your answer.

Lesson Check

Do you know HOW?

Solve each equation.

1. $w - 15 = 8.2$

2. $\frac{x}{3} = -30$

3. $2y - 1 = y + 11$

Solve each equation for k.

4. $r - 2k = 15$

5. $6k - 2z = 12$

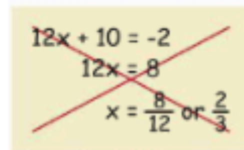
6. $4k + h = -2k - 14$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. Vocabulary Explain what it means to find a solution of an equation.

8. Reasoning Suppose you solve an equation and find that your school needs 4.3 buses for a class trip. Explain how to interpret this solution.

9. Error Analysis Find the error(s) in the steps shown.



Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Solve each equation. See Problem 1.

10. $h - 12 = 6$ **11.** $-\frac{x}{3} = 27$ **12.** $4t = 48$ **13.** $22 + r = 36$

Solve each equation. Check your answer. See Problem 2.

14. $7w + 2 = 3w + 94$ **15.** $15 - g = 23 - 2g$ **16.** $43 - 3d = d + 9$

17. $5y + 1.8 = 4y - 3.2$ **18.** $6a - 5 = 4a + 2$ **19.** $7y + 4 = 3 - 2y$

20. $5c - 9 = 8 - 2c$ **21.** $4y - 8 - 2y + 5 = 0$ **22.** $6(n - 4) = 3n$

23. $2 - 3(x + 4) = 8$ **24.** $5(2 - g) = 0$ **25.** $2(x + 4) = 8$

Write an equation to solve each problem. See Problem 3.

26. Bus Travel Two buses leave Houston at the same time and travel in opposite directions. One bus averages 55 mi/h and the other bus averages 45 mi/h. When will they be 400 mi apart?

27. Aviation Two planes left an airport at noon. One flew east and the other flew west at twice the speed. After 3 hours the planes were 2700 mi apart. How fast was each plane flying?

28. Geometry The length of a rectangle is 3 cm greater than its width. The perimeter is 24 cm. What are the dimensions of the rectangle?

Determine whether the equation is *always*, *sometimes*, or *never* true. See Problem 4.

29. $5x + 3 - 2x = 7x + 3$ **30.** $2(5x + 4) = 10x + 6$

31. $\frac{2}{3}x + 4 = 2x$ **32.** $6x - 12 + 2x = 3 + 8x - 15$

Solve each formula for the indicated variable.

See Problem 5.

33. $A = \frac{1}{2}bh$, for h **34.** $s = \frac{1}{2}gt^2$, for g **35.** $V = lwh$, for w **36.** $I = prt$, for r

Solve each equation for x.

37. $ax + bx = c$ **38.** $\frac{x}{a} - 5 = b$ **39.** $\frac{x-2}{2} = m + n$ **40.** $\frac{2}{5}(x+1) = g$

B Apply

Solve each equation.

41. $0.2(x+3) - 4(2x-3) = 0.9$

42. $12 - 3(2w+1) = 7w - 3(7+w)$

43. $(m-2) - 5 = 8 - 2(m-4)$

44. $7(a+1) - 3a = 5 + 4(2a-1)$

45. Think About a Plan The measures of an angle and its complement differ by 22°. What are the measures of the angles?

- What is true about the sum of the measures of an angle and its complement?
- When modeling the problem with an equation, how can you algebraically represent that the two angle measures differ by 22°?

Solve each formula for the indicated variable.

46. $R(r_1 + r_2) = r_1r_2$, for R **47.** $A = \frac{1}{2}h(b_1 + b_2)$, for b_2 **48.** $S = 2\pi r^2 + 2\pi rh$, for h

49. $h = vt - 5t^2$, for v **50.** $v = s^2 + \frac{1}{2}sh$, for h **51.** $R(r_1 + r_2) = r_1r_2$, for r_2

52. Writing Suppose you write and solve an equation to determine the amount of money m you have in your bank account after several weeks. You find that $m = -36$. What does this solution mean?

53. Geometry T he measure of the supplement of an angle is 20° more than three times the measure of the original angle. Find the measures of the angles.

54. Find 4 consecutive odd integers with a sum of 184.

Solve each equation for x.

55. $c(x+2) - 5 = b(x-3)$ **56.** $a(3tx - 2b) = c(dx - 2)$ **57.** $b(5px - 3c) = a(qx - 4)$

58. $\frac{a}{b}(2x - 12) = \frac{c}{d}$ **59.** $\frac{3ax}{5} - 4c = \frac{ax}{5}$ **60.** $\frac{a-c}{x-a} = m$

Write an equation to solve each problem.

61. Swimming A city park is opening a new swimming pool. You can pay a daily entrance fee of $3 or purchase a membership for the 12-week summer season for $82 and pay only $1 per day to swim. How many days would you have to swim to make the membership worthwhile?

STEM 62. Rocket The first stage of a rocket burns 28 s longer than the second stage. If the total burning time for both stages is 152 s, how long does each stage burn?

63. Error Analysis Your friend says that the equations shown are two ways to write the same formula. Is your friend correct? Explain your answer.

$s = \frac{n}{n+1}$ $\frac{s}{s-1} = n$

Challenge

64. Assume that a, b, and c are integers and $a \neq 0$.

a. **Proof** Prove that the solution of the linear equation $ax - b = c$ must be a rational number.

b. **Writing** Describe the values of a, b, and c for which the solutions of $ax^2 + b = c$ are rational.

65. A tortoise crawling at a rate of 0.1 mi/h passes a resting hare. The hare wants to rest another 30 min before chasing the tortoise at a rate of 5 mi/h. How many feet must the hare run to catch the tortoise?

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 4

Look back at the information on page 3 about Mia helping her friend Cody after Cody's car runs out of gas. In the Apply What You've Learned in Lesson 1-2, you thought about the possible locations of Cody's car when it runs out of gas.

Suppose Cody runs out of gas between the gas station and the restaurant. Let x represent the distance between Cody's car and the gas station. Choose from the following numbers and expressions to complete the equation below. The equation represents the relationship among the distances Mia drives.

9	11	20	34
$2x$	x	$9 + x$	$x - 9$
$9 - x$	$x + 11$	$x - 11$	$11 - x$

a. ___?___ + **b.** ___?___ + **c.** ___?___ = **d.** ___?___

- a. from Mia's house to Cody's car
- b. from Cody's car to gas station, then back to Cody's car
- c. from Cody's car to the restaurant
- d. total distance Mia drives

61. Answers may vary. Sample:

$2(b - a) + 5(b - a)$
$= (2 + 5)(b - a)$ Distr. Prop.
$= 7(b - a)$ Add.
$= 7b - 7a$ Distr. Prop.

Lesson 1-4 pp. 26–32

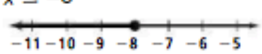
Got It? 1. $\frac{3}{2}$ **2.** -1 **3.** 40 m × 120 m **4. a.** never **b.** always **5. a.** $C = K - 273$ **b.** always

Lesson Check 1. 23.2 **2.** -90 **3.** 12 **4.** $k = \frac{1}{2}(r - 15)$ **5.** $k = \frac{1}{3}(z + 6)$ **6.** $k = -\left(\frac{1}{6}\right)(h + 14)$ **7.** To find a solution of an equation means to find the value of the variable that makes the equation true. **8.** Four buses are not enough. The number of buses must be a whole number, so round the number of buses to 5. **9.** The 2nd line is incorrect; subtract 10 from both sides: $12x = -12x = -1$

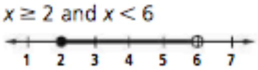
Exercises 11. -81 **13.** 14 **15.** 8 **17.** -5 **19.** $-\frac{1}{9}$ **21.** $\frac{3}{2}$ **23.** -6 **25.** 0 **27.** 300 mi/h; 600 mi/h **29.** sometimes **31.** sometimes **33.** $h = \frac{2A}{b}$ **35.** $w = \frac{V}{\ell h}$ **37.** $x = \frac{c}{a + b}$, $a \neq -b$ **39.** $x = 2(m + n) + 2$ **41.** 1.5 **43.** $\frac{23}{3}$, or $7\frac{2}{3}$ **45.** 34° and 56° **47.** $b_2 = \frac{2A}{h} - b_1$ **49.** $v = \frac{h + 5t^2}{t}$ **51.** $r_2 = \frac{Rr_1}{r_1 - R}$ **53.** 40°, 140° **55.** $x = \frac{3b + 2c - 5}{b - c}$, $\frac{b^\circ}{c}$ **57.** $x = \frac{4a - 3bc}{aq - 5bp}$, $5bp \neq aq$ **59.** $x = \frac{10c}{a}$, $a \neq 0$ **61.** Let c = number of swim days; $3c = 82 + c$; 41 days **63.** No; $n = \frac{s}{1 - s}$ ***not*** $\frac{s}{s - 1}$ **65.** 264 ft

Lesson 1-5 pp. 33–40

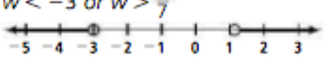
Got It? 1. $\frac{x}{3} \leq 15$
2. $x \leq -8$



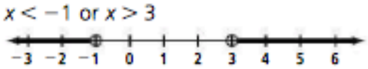
3. more than 32 songs **4.** always
5. a. $x \geq 2$ and $x < 6$



b. sometimes; The compound inequality is true when $x = 5$ and not true when $x = 7$.
6. a. $w < -3$ *or* $w > \frac{8}{7}$

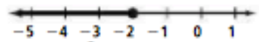


b. $x < -1$ or $x > 3$



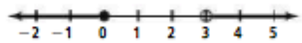
Lesson Check 1. $R \geq J$ **2.** $w \geq 40$ and $w < 74$

3. $x \leq -2$



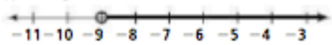
4. $1 < x < \frac{9}{5}$

5. $x \leq 0$ or $x > 3$

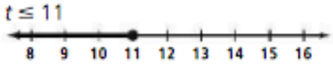


6. Answers may vary. Sample: $5 < 6$, but $-5 > -6$.
7. The transitive, addition and subtraction properties of inequality are similar to the properties of equality. The multiplication and division properties differ. Multiplying or dividing each side of an inequality by a negative number reverses the direction of the inequality symbol.
8. Answers may vary. Sample: $3x + 5 < 3(x + 5)$
9. No; Answers may vary. Sample: $2x < x + 1$ and $x + 1 > 3$

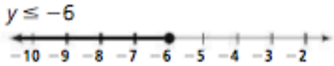
Exercises 11. $8x \geq 25$ **13.** $\frac{x}{12} \leq 6$
15. $k > -9$



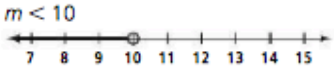
17. $t \leq 11$



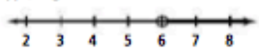
19. $y \leq -6$



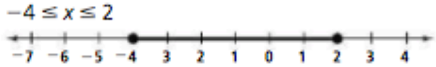
21. $m < 10$



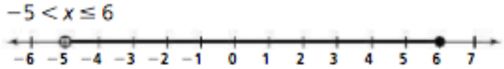
23. $w > 6$



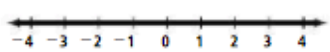
25. The longest side is less than 21 cm. **27.** at most 40 students **29.** always **31.** never **33.** sometimes
35. sometimes
37. $-4 \leq x \leq 2$



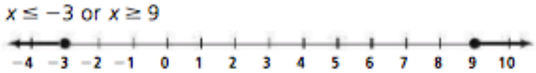
39. $-5 < x \leq 6$



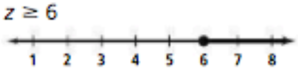
41. all real numbers



43. $x \leq -3$ or $x \geq 9$



45. $z \geq 6$



47. $x \geq -48$